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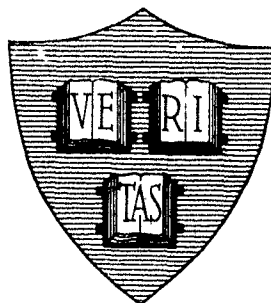
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Office of Naval Research

Contract N50RI-76 • Task Order No.1 • NR-372-012

FIELD EMISSION FROM GERMANIUM IN A
MÜLLER FIELD EMISSION MICROSCOPE



By

Frederick G. Allen

December 15, 1955

Technical Report No. 237

Cruft Laboratory
Harvard University
Cambridge, Massachusetts

Office of Naval Research

Contract N5ori-76

Task Order No. 1

NR-372-012

Technical Report

on

Field Emission from Germanium in a

Müller Field Emission Microscope

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Frederick G. Allen

December 5, 1955

The research reported in this document was made possible through support extended Cruft Laboratory, Harvard University, jointly by the Navy Department (Office of Naval Research), the Signal Corps of the U. S. Army and the U. S. Air Force, under ONR Contract N5ori-76, T.O. 1.

Technical Report No. 237

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I

Background

Many properties of single-crystal surfaces can now be directly observed, with high resolution under conditions of atomic cleanliness, by the use of the Müller-type microscope.^{1, 2} Metals used as the emitter tip of the microscope have been limited chiefly to those of high melting temperatures such as tungsten, molybdenum and tantalum, since they are easily cleaned of all contaminants, as well as smoothed and rounded by simply heating to temperatures near the melting point.

One of the aims of the present study was to attempt the use of single-crystal germanium tips as emitters in this type of microscope, in the hope of studying the work function, its variation with crystal direction, and gas contamination processes on a clean germanium surface. In 1951 Apker and Taft³ used sharply pointed single crystals of CdS and CdSe as field emitters in a Müller tube. While they were unable to produce the symmetrical emission patterns characteristic of metals, they did observe a very high sensitivity of the emission to illumination of the tip. The large increase in emission current with light was attributed to the increase in field strength at the surface caused by the reduction in voltage drop along the tip, and a readjustment of field configuration at the tip when the light increased the tip conductivity. Brock and Taylor⁴ attempted the use of both germanium and silicon tips in a Müller microscope during 1952. They worked out techniques for cutting, etching and mounting the tips,

and obtained electron microscope profiles demonstrating that at least some of their tips were smoothly rounded. They obtained field emission from about 30 tips in all, but were able to find no reproducible, symmetrical patterns.

Clarke⁵ has had some success in evaporating germanium onto a tungsten field emission tip in a Müller tube. When the tungsten tip was heated, the germanium was seen to migrate about on the surface and form clusters. In some cases it was believed that small crystallites of germanium built up on the tungsten and became field-emitter tips themselves since new symmetrical emission patterns, smaller and of different orientation from the tungsten patterns, now appeared, and at voltages below those required for the tungsten pattern itself. To our knowledge, this is the only claim made thus far to the production of symmetrical field-emission patterns from germanium.

II.

Apparatus and Techniques

(a) Etching Germanium Tips

While germanium tips were prepared by various methods, only that procedure used in the best-controlled study will be described here. A small rod of germanium 0.030 in. square in cross section and cut with a (110) axis, was waxed into a closely fitting hole in a brass rod. With this held in the chuck of a high-speed lathe, one end was turned down to a tapered conical tip with emery paper. (A sharp point was not necessary, but a round cross section of small diameter was found to remain round when etched into a tip, preventing "chisel edges" that sometimes occurred otherwise.) This tip was then etched electrolytically in a CP-4* solution. The end was lowered about 1 mm below the surface, a-c current of about 50 ma was used, and the tip was withdrawn gradually during the process. (The etching rate was greatest at the liquid surface.) A total etching time of one

- - - - -

*CP-4 solution: 25 cc HNO₃ conc.; 15 cc HAc, glacial; 15 cc HF conc.; several drops of bromine.

to two minutes, interrupted by low-powered microscopic examinations at about 20-second intervals, usually produced a smoothly tapered tip of a high polish with radius of curvature of from 10^{-4} to 10^{-6} cm. (It is believed that the high polish results from a sort of electropolishing action at the large current densities used — on the order of 10 amp/cm^2 .)

(b) Microscopic Examination of Tips.

The highest magnification found feasible for optical microscopic examination of the etched germanium tips was about 750 X.* This was not sufficient to resolve the surface profile of the tips sharply, so electron microscope profiles were taken at 3500X and 7000X magnification, using an M.I.T.-built microscope similar to the R.C.A. type U.M.T. The tip, 3/16 in. in total length, was waxed into the sample holder at right angles to the electron beam, which traveled through a 1/8 in. diameter hole along the axis of the 3/8 in. diameter sample rod. Magnification was computed for each tip from the image size of a fine 0.0003 in. diameter tungsten wire mounted in the holder close beside the tip. The large depth of focus of the electron microscope (on the order of 5.0×10^{-3} cm) and its high resolving power (down to approximately 30 Å), make it ideally suited for the examination of these tips.

(c) Experimental Tube.

The germanium tips were held in a 0.010 in. tantalum wire clip, shown in Fig. 1. The heavy supporting members were of tungsten covered with pyrex to eliminate outgassing problems. This assembly was sealed into the neck of a Müller tube made from a 500 cc spherical pyrex flask, with the germanium tip located at the center. The outer half of the sphere was made the anode and kept at uniform potential by the transparent tin oxide coating described in an accompanying technical report (T. R. No. 238). A thin (about $2\text{-}5 \text{ mg/cm}^2$) layer of zinc sulfide or zinc silicate phosphor was then sprayed into the tube suspended in a nitrocellulose binder, using a

* While higher magnifications can be achieved with optical microscopes, their resolution limit is on the order of the wavelength of visible light, which is comparable to the size of these tips.

paint-spray gun and rotating the tube slowly all the while with a motor drive. The binder was then baked off at about 400°C.

A tungsten filament was introduced in the tube through a sidearm to ionize argon by an electron stream when using ion bombardment to clean the tip, or to heat the tip by electron bombardment. The tip could also be brought to a red heat by passing current through the tantalum supporting wires.

(d) Electrical Circuitry.

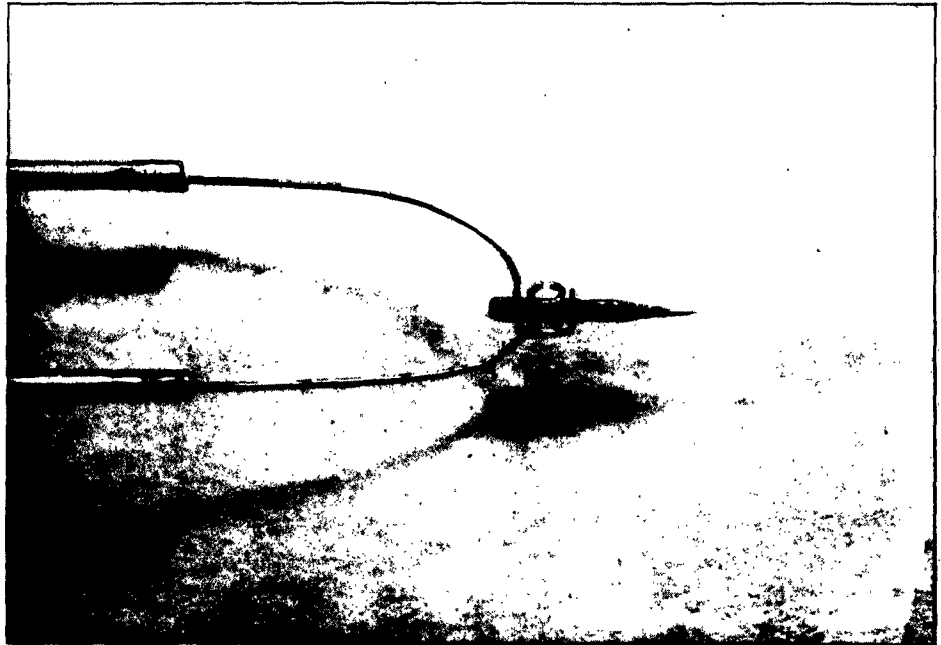
Voltages of 2000 to 12,000 volts were applied to the anode feed-through from a line frequency rectifier (voltage-doubler) unit. Voltages from 5,000 to 20,000 volts were obtained with an r-f television-type power supply, designed and built for this project to provide stabilized voltages of from a few thousand to 30,000 volts at up to 150 ma. Anode voltage was usually read with an R.C.A. high-voltage probe and a Volt-Ohmyst voltmeter. (A dropping resistor of from 2000 to 10,000 megohms was usually used between high-voltage supply and anode to prevent loss of emitter tips through sudden excessive current.) Total emission current was measured with an R.C.A. Ultrasensitive Microammeter placed between ground and the emitter tip.

III.

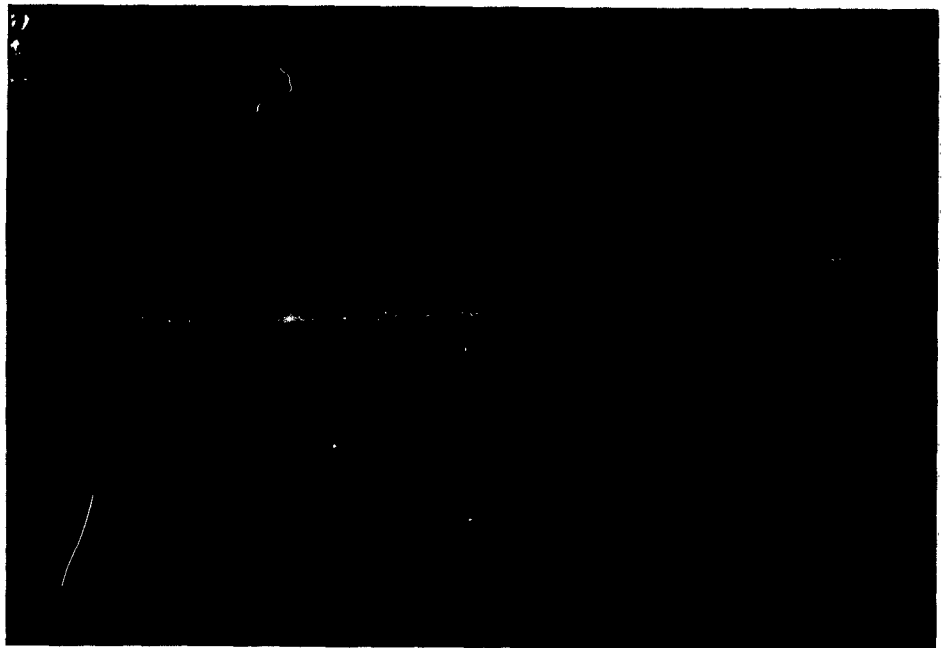
Experimental Results

(a) The Nature of Tips Produced.

Figure 1-b shows an optical microscope photograph of an etched germanium tip (No. 2), at 750X magnification. Electron microscope profiles of tips No. 1 and No. 2 are reproduced in Figs. 2-a and b. Profiles taken at right angles to those reproduced here showed about the same radius of curvature at the tip, so that the tips were not "chisel-edged." It is seen that at least in the case of tip No. 2, any roughness on the surface is extremely small compared to the radius of curvature of the tip. For these tips, radii of curvature measured from the electron microscope profiles were $1.2 \pm 0.2 \times 10^{-5}$ cm for tip No. 1, and $7.3 \pm 1.0 \times 10^{-5}$ cm

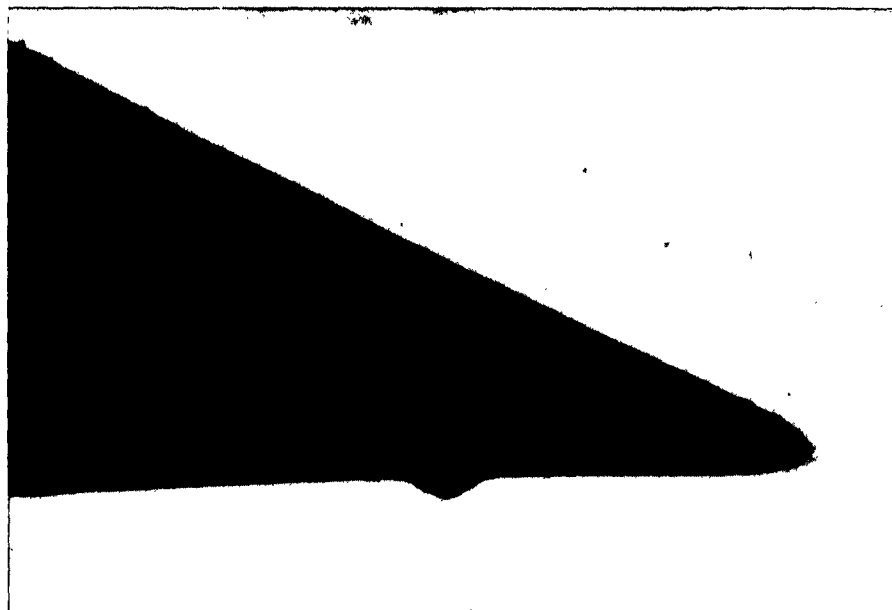


a. MOUNTING OF GERMANIUM TIP

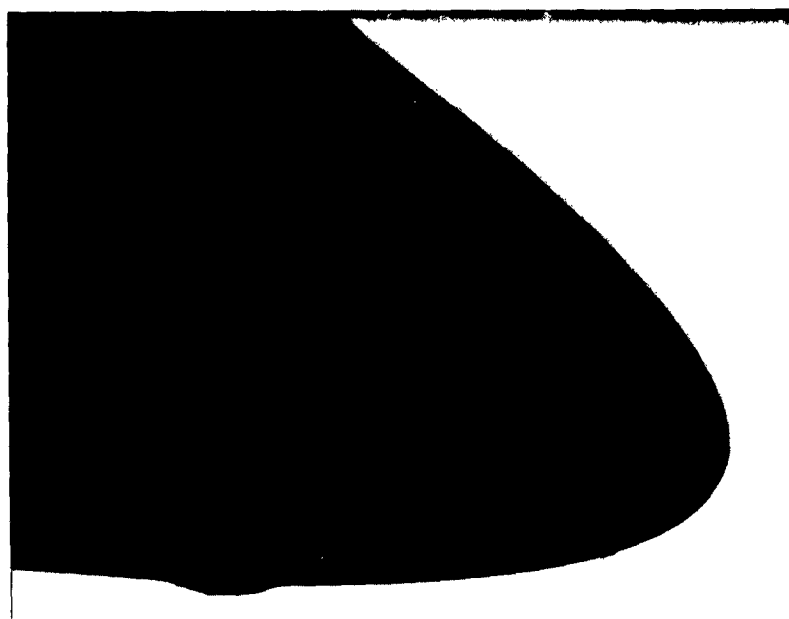


b. OPTICAL MICROSCOPE PHOTOGRAPH OF TIP #2.750 X

FIG. 1 GERMANIUM FIELD EMISSION TIP AND MOUNTING



a TIP #1 MAGNIFICATION 7000 X



b. TIP #2 MAGNIFICATION 8000 X

FIG. 2 ELECTRON MICROSCOPE PROFILES OF
GERMANIUM FIELD EMISSION TIPS

for tip No. 2.

(b) The Nature of the Field Emission
Observed from Germanium.

Field emission producing visible spots on the fluorescent screen was obtained with all the germanium tips used at initial anode voltages of from 1,500 to 10,000 volts. After the onset of emission, current increased exponentially with anode voltage. Figure 3-a shows data on total emission current I_T vs anode voltage V_a taken on the tip No. 2 which is typical of data of this kind taken on all the germanium tips. Curve b in this figure is the same data plotted in the form

$$\log_{10} I_T \text{ vs } 1/V_a$$

It is seen that the latter plot is nearly a straight line* with negative slope, or

$$\log I_T = A - \frac{b}{V_a} \quad (1)$$

This gives the relationship between I_T and V_a as

$$I_T = A' e^{-\frac{b}{V_a}} \quad (2)$$

Most of the scatter of the data was caused by a slight instability of the emitting surface. Some emitting spots would appear or disappear, or change intensity suddenly.**

The only difference in the data taken on different germanium points, or on the same points after different treatments, lay in the intercept, A, and the slope b.

*The apparent departure of the $\log I_T$ vs $1/V_a$ data from linearity at the higher-current end in this figure is probably not significant, since linearity in this plot was often preserved in other data to current density levels that were orders of magnitude higher than the maximum in this run.

**This flickering of spots in the field emission pattern may be related to the somewhat similar flickering in the avalanche breakdown current across a p-n junction studied recently by McKay and others at the Bell Laboratories. (Private communication, K. G. McKay.) However, it is believed that the limiting process in the field emission was normally penetration of the potential barrier at the surface, as evidenced by the true exponential current-voltage relationship.

Data taken on clean tungsten field emitter tips in this tube gave the same type of results as these, again differing only in slope and intercept from the curve in Fig. 3-b. Stability of emission for the tungsten was better than for the germanium and there was somewhat less scatter in the data.

While the initial emission from good tungsten tips was sometimes a definitely symmetrical pattern even before high-temperature cleaning had begun, in no case – even after exhaustive cleaning processes – was a clear-cut symmetrical pattern observed from a germanium tip. The closest approach to success is shown in Fig. 4, where photographs of the field emission pattern from germanium tip No. 2 are shown (a) after heating several times to approximately 450°C , and (b) after heating to approximately 550°C . The small light spots which appear in different positions on the two patterns are due to contaminant molecules or clusters of molecules. These appeared in all the forms – cloverleaves, two-leaved spots, single spots, etc. – described in detail by other workers with the field emission microscope,⁶ and were far more persistent with germanium tips than with tungsten.

Aside from these random spots in Fig. 4, there is a remaining emission from the germanium substrate itself that is essentially unchanged in the two pictures. The crystallographic axes, known to within approximately 10 degrees from X-ray data on the tip before mounting, are indicated. For the germanium (diamond) lattice, there should be a four-fold symmetry in this view, i.e., reflections about the two heavy axes shown.* There is a suggestion of the expected symmetry, particularly in the structure below the [100] axis and on either side of the [110] axis; it is possible that a further cleaning of the surface of the proper sort might have developed the remaining portions of the pattern. After these pictures were taken, a series of argon bombardments** was carried out on the tip, but these introduced more random spots and gradually eradicated what semblance of

- - - - -
*The [100] axis is that shown as nearly horizontal, and the [110] as nearly vertical.

**The argon bombardment technique used here was similar to that reported by Farnsworth et al.⁷ Positive argon ions were accelerated to the surface through 300 to 500 volts, using an argon pressure of $\sim 10^{-4}$ mm Hg, with a current density of $\sim 10 \mu\text{a}/\text{cm}^2$ for a few minutes. (See also Technical Reports 236 and 237.)

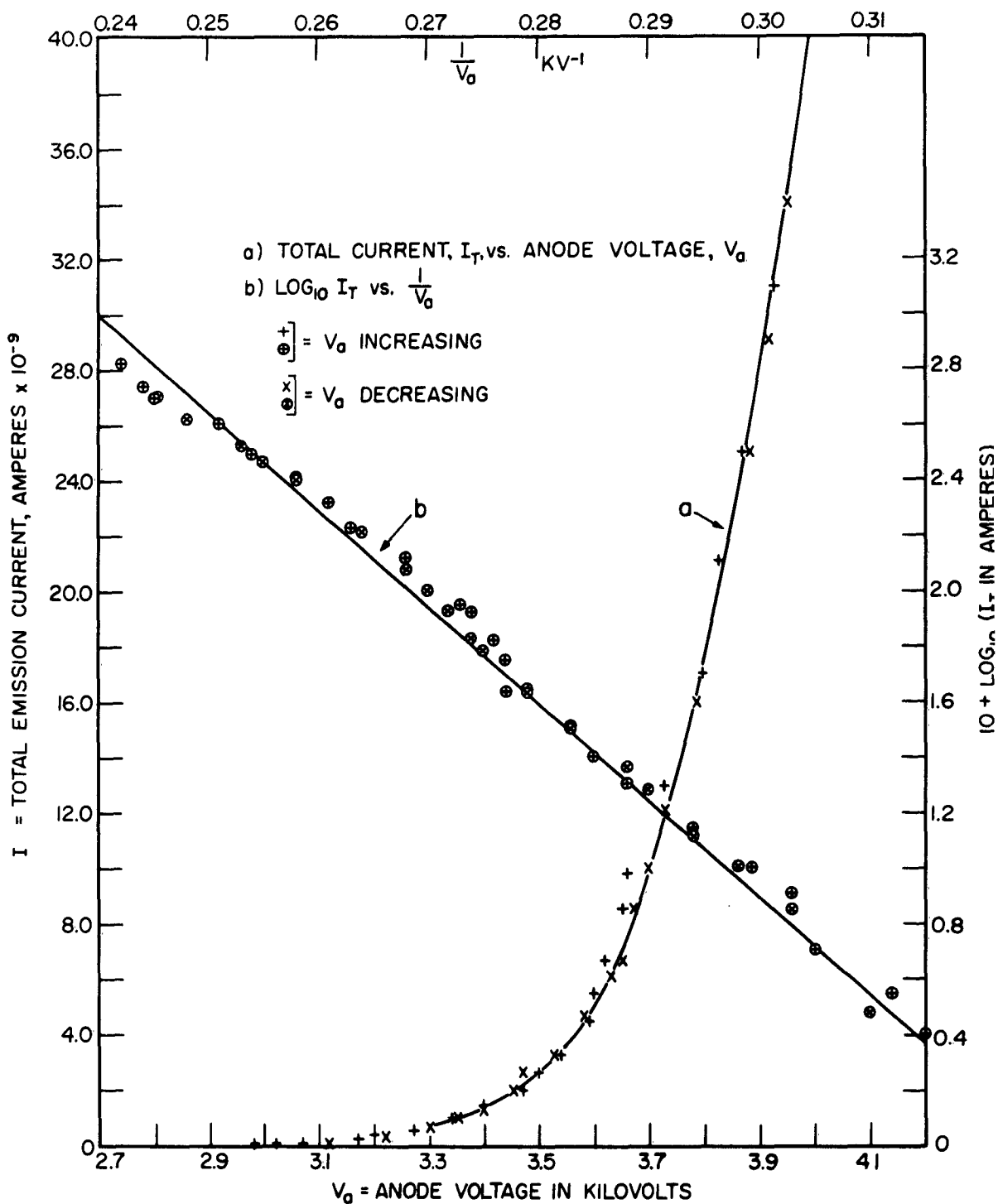
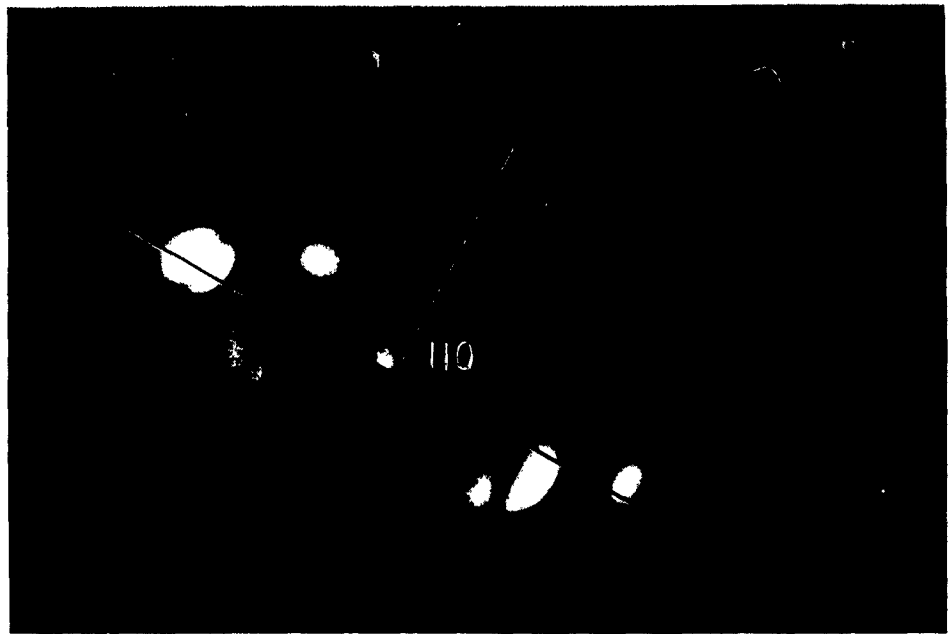


FIG. 3 FIELD EMISSION FROM GERMANIUM TIP #2
(AFTER SEVERAL ARGON BOMBARDMENTS)



a. TIP PREVIOUSLY HEATED TO 450° C



b. TIP PREVIOUSLY HEATED TO 550° C

FIG. 4 FIELD EMISSION PATTERNS
FROM GERMANIUM TIP #2

symmetry there was after the mild heating alone.

Electron microscope profiles taken of tips No. 1 and No. 2 after they had been used as field emitters and cleaned by argon bombardment, showed that bombardment, if excessive, can sputter away the entire tip (as occurred with No. 1). Even if carried out mildly, it appears to leave the surface somewhat roughened, or at least tends to accentuate and sharpen any slight protuberances originally present. *

Other cleaning treatments that were tried without success included:

- (i) Bombardment with hydrogen ions.
- (ii) Heating to high temperatures in hydrogen.
- (iii) Heating to high temperatures both by electron bombardment and by use of a resistive heating current.

While all of these treatments could be made to alter the pattern of spots on the screen, and some of them, particularly (ii) and (iii), gave some evidences of cleaning the tip with a resultant increase in emission for a given voltage, none of them produced a clear-cut characteristic pattern.

In retrospect, it is felt that in order to succeed (if success is possible) a repetition of this experiment should include:

First, the production of a relatively large tip by etching and its examination in an electron microscope to make certain of a highly smooth surface. These requirements were satisfied by tip No. 2.

Second, prolonged outgassing of the tip (about 1 hour) in a vacuum of 10^{-9} mm Hg, or better, at a temperature of $700 - 800^{\circ}\text{C}$.

Evidence from the C.P.D. studies on germanium surfaces (Technical Report 236) indicates that in many cases this should produce a nearly clean surface without requiring argon bombardment. The latter seems to be too rough a treatment for this experiment.

- - - - -

*Müller has explained this sharpening action, observed by him on sputtering tungsten tips, as due to the inability of the heavy ions to follow the electric field lines at the tip, causing them to strike and erode away material just below the point.

(c) Further Observations on Field Emission from Germanium

i. Doping: Field emission has been obtained from intrinsic and both n- and p-type germanium tips. (The p-type was 4×10^{16} gallium atoms/cm³; the n-type samples included one lot of 0.3 ohm-cm, 6×10^{15} donors/cc, and another lot of 0.06 ohm-cm, 3×10^{16} donors/cc.) While data on tip size were not taken, the general emission characteristics for the various types of germanium were not strikingly different.

ii. Temperature dependence: The field emission current at a given voltage from germanium tips that had previously been cleaned by heating to 500 - 700°C did not vary appreciably as the germanium temperature was raised from room temperature to about 350°C. At higher temperatures, emission tended to rise in an irreversible manner. Activity and sudden changes in the emission pattern at increased temperatures suggested that changes in the surface structure were taking place. Since it is known that germanium softens above 500°C,⁸ it seems probable that the large tensile forces exerted by the electric field produced a building-up of protuberances on the tip, an effect which has been observed at elevated temperatures by other workers on metal tips. For example, both Benjamin and Jenkins,⁹ and Gomer¹⁰ reported such a build-up process with applied field at as low as 200 - 300°C for nickel field emitter tips. (Nickel annealing temperature is also close to 500°C.) However, in the case of metal tips, the protuberances form first in well-defined crystallographic positions and if prevented from going on to cause failure, produce sharp but still symmetrical patterns. Protuberances forming on the germanium tips seem to have been small, randomly located needle-like tips, producing intensely sharp emission spots on the screen. On some tips examined after use as emitters, such needle-like structures were found on the originally smooth tip surface. A curious effect, which can also be explained on the assumption of emission from these needle-like "whiskers," was the following: An extremely small, intense spot on the screen would occasionally move in a rapid oscillatory fashion while the remainder of the pattern was at rest.

In the case of metal tips, heating to near the melting point with no field applied apparently allows the surface tension to restore the tip to a smoothly rounded form. Many attempts were made to perform such a smoothing

on germanium tips at high temperatures, but only occasionally did a rough pattern of spots become more uniform. Frequently, the heating introduced new irregular emission regions. (It is possible that a better-controlled heating at near the melting point might still produce smoothing.)

iii. Thermionic emission from germanium: In one case the temperature was purposely raised until the germanium tip melted and coalesced into a drop on the tantalum supports. Upon heating this "drop" up from room temperature with a field applied, a strong emission illuminated the phosphor as the germanium became glowing red. As the temperature was raised further, the surface of the drop as shown by the pattern became visibly mobile, and wavy circles of bright emission traveled slowly out from the center of the tip back toward the equator. On cooling, several sharp tips built up on the surface of the drop, which continued to field-emit at low anode voltages (5 kv) at room temperature. No further data were taken at high temperatures to show that thermionic emission of electrons was involved. Emission current densities from the ~ 1 mm diameter germanium sphere were estimated at 10^{-5} to 10^{-4} amp/cm².

iv. Mechanical strength of germanium tips: Germanium field emitter tips withstood calculated electric field strengths of at least 2.0×10^7 volts/cm without suffering physical damage. Since tensile stress normal to the surface is given by $\frac{E^2}{8\pi}$ dynes/cm² (E in stat-volts/cm), we conclude that small single crystals of germanium have a tensile strength of at least 175 kg/cm². *

v. Failure of germanium tips: Failure of tips always seemed to occur together with excessive emission from some small spot on the surface. At a constant anode voltage, this emission would grow more and more intense, while the remainder of the pattern remained unchanged, until finally it would disappear suddenly, often leaving new emission spots around it, as though from the rough edge of a crater. Frequently the emitting spot scintillated violently during its growth. The same scintillation was observed

*Recent work on germanium single crystal whiskers, (virtually no dislocations) by Pearson, Read and Feldman at the Bell Laboratories indicates that a germanium crystal may approach fracture stresses of 120,000 kg/cm² when in this form. Estimated polycrystalline tensile strength is ~ 500 kg/cm². (Private communication, G. L. Pearson.)

over all emitting regions whenever an ionic bombardment was taking place.

Possible mechanisms for the failure are:

(a) Local resistive heating of the small emitting region due to high-emission current density, which allows a building up and sharpening of a tip due to the high applied field. This, in turn, enhances the field and emission still further until the tip pulls off.

(b) Intense local ionic bombardment of the tip may either assist in the heating process or may erode away the neck of the tip and sharpen it. The ions are produced by collision of emitted electrons with residual gas molecules in the tube.*

Rough calculations on probable magnitudes of the above effects indicate that the resistive heating of the tip probably far outweighs the local heating due to ionic bombardment, at least until germanium is vaporized. Maximum calculated current densities for cases just before a tip "exploded" lie between 10^5 and 10^8 amp/cm². Dolan, Dyke and Trolan¹¹ have reported current densities of from 10^7 to 10^8 amp/cm² for tungsten tips just before failure. The resistivity of germanium** is about one order of magnitude greater than that of tungsten when both are near the melting point, while its thermal conductivity is about one third that of tungsten and its specific heat per unit volume roughly the same. It seems likely, then, that germanium tips, or small emitting regions thereof, can be brought to their relatively low softening or vaporizing temperatures at considerably lower emission densities than those causing failure for tungsten.

vi. Dependence of field emission from germanium upon illumination:

Several field emitter tips were illuminated during emission to find whether or not the change in contact potential with light, $(\Delta C.P.)_L$, described in Technical Report 236, persisted at high field strengths and caused noticeable

*This same effect causes field emission patterns to contaminate more rapidly when the tube is emitting than when it is not; this ionic contamination should become more important than neutral gas molecule contamination at emission currents greater than 0.1 to 1.0 μ a.

**The effect of enhanced conductivity at the surface under high positive applied fields has been neglected here.

effects. Except for a slow change in emission upon illumination found with two tips, which was shown to be a temperature-induced effect (due presumably to unclean surfaces), and a very small instantaneous increase in emission for some tips which was roughly independent of applied field* and could be attributed to direct photoemission from the germanium sample or its supporting wires, no other effects on emission were noted.** (Light intensities were equal to or greater than those used in the $(\Delta C.P.)_L$ study.)

IV.

Discussion of Results for Field Emission from Germanium

(a) Effect of Applied Field on Energy Levels at the Surface for No Surface States. (See Fig. 5.)

In order to relate the field emission from germanium to that from metals, which is fairly well understood, we must first predict the nature of the energy structure at the surface of a semiconductor at fields high enough to cause appreciable tunneling of electrons through the surface barrier.*** We first consider the situation at very low emission current levels where thermal equilibrium may still be assumed. Further, we shall treat the case for the simple surface without surface states, and then predict the modifications they introduce.

Starting with Poisson's equation in a one-dimensional surface region of dielectric constant χ , we have

$$\frac{d^2\psi(x)}{dx^2} = -\frac{4\pi\rho(x)}{\chi} \quad (3)$$

*If the change in emission were due to a change in work function upon illumination, $(\Delta C.P.)_L$, one would expect this change to increase rapidly (in absolute value) with field. (See the form of the Fowler Nordheim equation, section 4-g.)

**Due to the high electron density produced at the surface by the field, recombination rates may have been so high as to make net pair concentration, and hence $(\Delta C.P.)_L$, negligible.

***A recent paper (in the press) by Wannier and Herring suggests that on theoretical grounds the forbidden energy gap of a semiconductor may be expected to disappear at very high electrical field strengths. Such effects are neglected in this discussion.

Multiplying by $2 \frac{d\psi}{dx}$, changing the independent variable from x to ψ on the right-hand side, and integrating, we have

$$\left(\frac{d\psi}{dx}\right)^2 \int_{x_1}^{x_2} dx = -\frac{8\pi}{K} \int_{\psi_{x_1}}^{\psi_{x_2}} \rho(\psi) d\psi \quad (4)$$

If we now choose x_1 to be at the surface itself and x_2 to be in the interior where no field exists, we find

$$F^2 = 8\pi K \int_{\psi_s}^{\psi_0} \rho(\psi) d\psi \quad (5)$$

where F is the applied field just outside the surface.

If (5) is applied to a metal, where $\rho(\psi - \psi_0)$ is very large for even small values of $(\psi - \psi_0)$, it is found that field strengths F sufficient to cause field emission (approximately 2×10^7 volts/cm) cause a depression in the energy levels at the surface of only the order of hundredths of an eV. Electron density is high enough so that a small change in the first one or two atomic layers produces a charge sufficient to cancel the applied field.

For a semiconductor with N_d and N_a completely ionized donor and acceptor levels per cm^3 , we have (12)

$$\rho(\psi) = e[N_d - N_a + 2n_i \sinh \frac{e(\phi - \psi)}{kT}] \quad (6)$$

Substituting (6) into (5), integrating and using $\rho_0 = 0$ in the bulk, we have

$$\frac{F^2}{8\pi e K} = -(\psi_s - \psi_0)N' - \frac{2n_i kT}{e} \left(\sqrt{1 + \left(\frac{N'}{2n_i}\right)^2} - \cosh \frac{e(\psi_s - \phi)}{kT} \right) \quad (7)$$

where $N' = N_d - N_a$.

When the value of the electrostatic potential of the Fermi level

$$\phi = \psi_0 - \frac{kT}{e} \sinh^{-1} \frac{N'}{2n_i} \quad (8)$$

is substituted in Eq. (7), it can be solved graphically for various field strengths, F , and dopings, N' , to find the resulting surface potential displacement $(\psi_s - \psi_0)$. For a positive applied field, negative charge is drawn

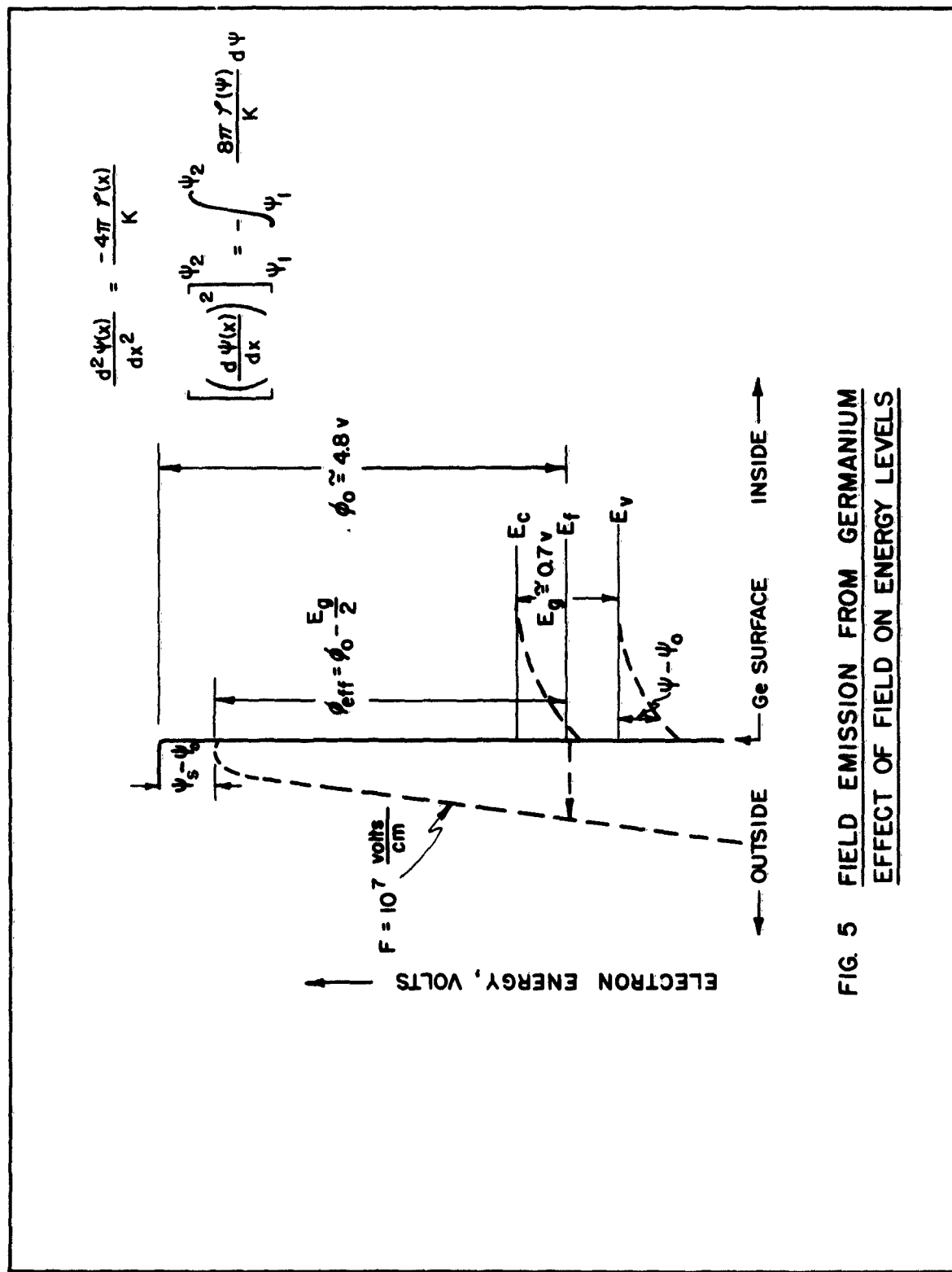


FIG. 5 FIELD EMISSION FROM GERMANIUM
EFFECT OF FIELD ON ENERGY LEVELS

to the surface and the energy bands are depressed as shown in Fig. 5. Solutions of (7) indicate that for all ranges of doping normally encountered in germanium (N' running to $\pm 10^{19}/\text{cm}^3$), the conduction band level E_c is brought down to within a few kT of the Fermi level at applied fields F of between 5×10^6 and 10^7 volts/cm. The form (6) is no longer valid when the donor energy level E_d , or the conduction band level, E_c , is within a few kT of the Fermi level, when the Boltzmann approximations must be replaced by the actual Fermi function. While this can be done, it is tedious, and for the present work it will be sufficient to continue the calculation after E_c at the surface has passed below E_f and the filled parabolic energy state density, found in the conduction band for metals, results:

$$N(E) = \frac{4\pi}{h^3} (2m)^{3/2} (E - E_c)^{1/2}, \quad E < E_f \quad (8)$$

We can now neglect the charge in states lying above E_f , compared to that below, so that the net charge density at a point inside the surface where E_c falls below E_f by the amount

$$e \psi'(x) = E_f - E_c(x) \quad (9)$$

becomes

$$\rho(x) = -e \int_{E=E_c}^{E=E_f} N(E) dE = \frac{8\pi e}{3h^3} (2m)^{3/2} \psi'(x)^{3/2} \quad (10)$$

Now, formally, (4) can be broken up into different regions, in each of which a given charge density holds, as

$$\left(\frac{d\psi}{dx}\right)^2 \int_{x_1}^{x_2} + \left(\frac{d\psi}{dx}\right)^2 \int_{x_2}^{x_3} + \dots = -\frac{8\pi}{\chi} \left[\int_{\psi_{x_1}}^{\psi_{x_2}} \rho_{1-2}(\psi) d\psi + \int_{\psi_{x_2}}^{\psi_{x_3}} \rho_{2-3}(\psi) d\psi + \dots \right] \quad (11)$$

If we now let $\rho_{1-2}(\psi)$ be that defined by (10), and let x_1 = the surface, x_2 = the point where E_c crosses E_f , noting that $d\psi = -d\psi'$, the first terms from the right and left sides of (11) give

$$\begin{aligned}
 F^2 - F_{x_2}^2 &= -\frac{8\pi}{K} \frac{8\pi e}{3h^3} (2m)^{3/2} \int_{\psi'_s}^0 \psi'^{3/2} d\psi' \\
 &= \frac{(8\pi)^2 e}{3K h^3} (2m)^{3/2} \frac{2}{5} (\psi'_s)^{5/2}
 \end{aligned} \tag{12}$$

where ψ'_s is the value of $\frac{1}{e} (E_f - E_c)$ at the surface.

We now find that for values of ψ'_s of more than several kT , $F_{x_2}^2 \ll F^2$ so that the charge in the space-charge layer for $x > x_2$ can be neglected compared to that very close to the surface. Solutions of (12) for germanium then give the depression of E_c at the surface below E_f when this is more than several kT and the results connect smoothly with solutions of (7) for lower fields. The values obtained show that for all dopings except extreme p-type ($N' > 2 \times 10^{19}/\text{cm}^3$), E_c lies on the order of $2kT$ beneath E_f at the surface for an external applied field F of 10^7 volts/cm.

We now wish to estimate how large F must be, and hence how far the levels may actually be depressed at the surface, when field emission begins for germanium. The quantum-mechanical probability $D(W)$ that a free electron of normal kinetic energy W incident from one side upon a potential barrier of shape $U(x)$ shall penetrate the barrier and appear outside can be given by (13)

$$D(W) = e^{-\frac{2\pi\sqrt{8m}}{h} \int_{x_1}^{x_2} (U(x) - W)^{\frac{1}{2}} dx} \tag{13}$$

where x_1 and x_2 are the points at which the integrand vanishes. This penetrability of the barrier is less sensitive to its shape than to its height and thickness, and is an extremely rapidly varying function of both the latter. The applied field at the surface of a metal increases this penetrability for an electron of given normal kinetic energy (W) by (i) narrowing it, since the barrier decreases outside the surface with a slope of $-F$, (see Fig. 5), and (ii) by lowering the maximum height through the interaction of the field and image potential (Schottky effect).

To give an example, for an image force barrier having $U(x)_{\text{max}} - W = 4 \text{ eV}$, $D(W)$ varies from 10^{-23} at $F = 10^7$ volts/cm to $10^{-4.6}$ at

$F = 5 \times 10^7$ volts/cm. The number of electrons of normal kinetic energy incident on the barrier per second- cm^2 for a metal (electrons are assumed to be free) is (13)

$$N(W) \cong \frac{4\pi m}{h^3} (E_F - W) dW \cong 10^{29} (E_f - W) dW \quad (14)$$

The total resulting emission current density for field emission is

$$J = \int N(W) D(W) dW \quad (15)$$

Since field emission electrons are found to originate from within about 1 eV of the Fermi level for tungsten (14), we must have, for an appreciable J of 1 amp/ $\text{cm}^2 \cong 10^{19} \frac{\text{electrons}}{\text{cm}^2\text{-sec}}$

$$N(W) \cdot D(W) \Delta W \cong 10^{29} \cdot 1 \cdot D(W) \cdot 1 = 10^{19}$$

$$\text{or } D(W) \cong 10^{-10} \quad (16)$$

For the above values, then, field emission from a metal would first become appreciable at $F \cong 2 \times 10^7$ volts/cm.

The barrier cannot be very different for germanium from that for metals, since the work function, approximately 4.8 eV, is close to that for tungsten, and the image force is nearly that for a metal because the dielectric constant $K = 16$ is large enough so that $\frac{K-1}{K+1} \cong 1$. However, the depression of the energy bands at the surface by the applied field as just discussed for no surface states will decrease the barrier height with respect to the Fermi level an additional amount varying from approximately 0 for extreme n-type to approximately E_G for extreme p-type, at fields of $\sim 10^7$ volts/cm. Since this is not a drastic change in the work function we should still expect to find field emission first occurring for germanium at fields within a factor of 10 of 2×10^7 volts/cm.

We can conclude, then, that for no surface states, the germanium surface would be metallic or nearly so, before field emission occurred. Hence the Fowler-Nordheim treatment for field emission for metals, which results from integrating (15), assuming the emitted electrons originate from a parabolic conduction band filled to the Fermi level, should be expected to

apply qualitatively to germanium. (See section (g) for the equation.)

(b) Modifications Introduced by Surface States.

The presence of a large density of surface states near the Fermi level will, in general, make the surface potential more insensitive to applied external fields, since a larger change in surface charge now results from a given potential shift. We can carry out a simple calculation to determine how large a density of surface states, uniformly distributed at the Fermi level, is needed to make the surface potential insensitive to external applied fields on the order of those required for field emission. If the assumed constant density of states is η_s per cm^2 per energy increment, and if we neglect all surface charge other than that in the filled surface states below the Fermi level, the change in surface net charge produced by a surface potential displacement $\Delta\psi_s$ is just

$$\Delta\sigma_s = e\Delta\psi_s \eta_s \quad (17)$$

To neutralize the external field this charge must satisfy

$$F = 4\pi \Delta\sigma_s = 4\pi e \Delta\psi_s \eta_s \quad (18)$$

or

$$\Delta\psi_s = \frac{F}{4\pi e \eta_s}$$

If we now stipulate that $\Delta\psi_s$ remain less than 0.1 eV. when $F = 10^7$ volts/cm, we have

$$\begin{aligned} \eta_s &= \frac{F}{4\pi e \Delta\psi_s} \geq \frac{10^7/300}{4\pi (4.8 \times 10^{-10})(0.1)} \\ &= 5 \times 10^{13} \frac{\text{states}}{\text{cm}^2\text{-ev}} \end{aligned} \quad (19)$$

For states more densely located further from the Fermi level a higher value would be required.

We conclude that if surface states at the Fermi level have a density of approximately $\frac{1}{50}$ to $\frac{1}{10}$ of that of the surface atoms for germanium, little or no displacement of the surface potential would occur under field emission conditions. For the surface state case, then, the field emission would differ

from that for a metal in that the emitted electrons would probably originate from surface levels instead of from a partially filled conduction band. The density of these electrons at the surface, however, would still be within two orders of magnitude of one per surface atom, the latter figure being close to the number of conduction electrons present in a metal surface. Since an order of magnitude change in field emission current can easily result from a 5 or 10 per cent change in applied field, (see Fig. 3), this difference in density is probably not significant, and we should again expect results similar to those for a metal, provided the quantum mechanical transition from a surface energy state to a position outside the barrier is not radically different from that for a more nearly free electron in a conduction band.

Since the densities of surface states deduced from high-vacuum germanium surface studies discussed in Technical Report 236 are comparable to the figure found here to be required to prevent appreciable displacement of the energy levels by the applied field, the actual field emission case probably involves some level displacement, but also some emission from surface states.

(c) Field Strength at Tips for Emission.

We can estimate the minimum field strength required to produce field emission for germanium tips No. 1 and No. 2 from the electron microscope profiles (Fig. 2) and the value of applied anode voltage, V_a , at which emission began. Analyses of two profiles of tip No. 2 show it to be very close to a paraboloid of revolution with radius of curvature at the tip of approximately 7.3×10^{-5} cm. Exact electrostatic potential solutions can readily be obtained for a set of confocal paraboloidal equipotential surfaces, so that by choosing one surface to be the tip at ground potential and another larger one the anode at a high potential, we obtain one approximation to the field F at the tip surface for a given anode voltage V_a . Since the true anode is a sphere at uniform potential, the above approximation will give too low a value for the field. As a second approximation, we use the solutions for a sphere of radius a , mounted with its center at the apex of a supporting cone, of half-angle θ_0 , as the tip, surrounded by a larger

concentric sphere of radius b for the anode. If a is set equal to the observed minimum radius of curvature of the actual tip, b equal to the true anode radius, and θ_0 equal to the actual half angle of the tip base, this approximation will give too large a field. Hence, we have upper and lower bounds to the true field.

For the confocal paraboloidal surfaces Dyke and Trolan¹⁵ give the radial field strength at the vertex of the tip due to an applied anode voltage, V_0 as

$$F_0 = \frac{V_0}{a \ln b/a} \quad (20)$$

where a and b , the distances from focus to vertex of the smaller and larger surfaces, are set equal to the radius of emitter and anode respectively.

The value of field at the surface of an isolated small sphere of radius a inside a larger concentric sphere of radius b for $a/b \ll 1$, is

$$F'_0 = \frac{V_0}{a} \quad (21)$$

We now define the factor γ that relates F_0 for other geometries to that for the spheres, as

$$F_0 = \gamma F'_0 \quad (22)$$

so that for the paraboloidal case

$$\gamma_p = \frac{1}{\ln b/a} \quad (23)$$

For tip No. 2, $a = 7.3 \times 10^{-5}$ cm, $b = 5$ cm, so that

$$\gamma_{p,2} = \frac{1}{\ln 6.85 \times 10^4} = \frac{1}{11.5} \quad (24)$$

For the grounded sphere-on-orthogonal cone surrounded by a spherical anode at constant potential, V_0 , Hall¹⁶ gives the potential distribution as a function of radius, r , and angular displacement, θ , from the polar axis directed outward through the tip vertex as

$$V(r, \theta) = V_0 \sum_{s=0}^{\infty} \frac{I_s(\cos \theta_0)}{H_s(\cos \theta_0)} \frac{[(V/a)^{\nu_s} - (a/r)^{\nu_s+1}]}{[(b/a)^{\nu_s} - (a/b)^{\nu_s+1}]} P_{\nu_s}(\cos \theta) \quad (25)$$

$$\text{where } I_s(\cos \theta_0) = \int_0^{\theta_0} P_{\nu_s}(\cos \theta) \sin \theta \, d\theta$$

$$H_s(\cos \theta_0) = \int_0^{\theta_0} [P_{\nu_s}(\cos \theta)]^2 \sin \theta \, d\theta$$

θ_0 = half angle of cone

$P_{\nu_s}(\cos \theta)$ = Legendre polynomial of first kind, degree ν_s ; ν_s , $s = 0, 1, 2$, etc. = degree of Legendre polynomials which vanish at $\theta = \theta_0$. In general, ν_s is real but nonintegral.

To find the radial field strength near the surface of the tip on the polar axis, set $\theta = 0$, $P_{\nu_s}(\cos \theta) = 1$, and we have

$$F(r, 0) = -\frac{dV(r, 0)}{dr} = V_0 \sum_{s=0}^{\infty} \frac{I_s(\cos \theta_0) \left[\nu_s \frac{r^{\nu_s-1}}{a^{\nu_s}} + (\nu_s+1) \frac{a^{\nu_s+1}}{r^{\nu_s+2}} \right]}{H_s(\cos \theta_0) [(b/a)^{\nu_s} - (a/b)^{\nu_s+1}]} \quad (26)$$

Setting $r = a$ we find the field at the vertex of the tip to be

$$F_0 = -\left. \frac{dV(r, 0)}{dr} \right|_{r=a} = \frac{V_0}{a} \sum_{s=0}^{\infty} \frac{I_s(\cos \theta_0) [2\nu_s+1]}{H_s(\cos \theta_0) [(b/a)^{\nu_s} - (a/b)^{\nu_s+1}]} \quad (27)$$

so that for the sphere-on-orthogonal cone case

$$\gamma_s = \sum_{s=0}^{\infty} \frac{I_s(\cos \theta_0) [2\nu_s+1]}{H_s(\cos \theta_0) [(b/a)^{\nu_s} - (a/b)^{\nu_s+1}]} \quad (28)$$

Hall has tabulated values of I_s/H_s and ν_s , $s=0, 1, 2, \dots$, etc. for various half angles θ_0 . For tip No. 2, $\theta_0 \cong 12^\circ$, $b/a = 6.85 \times 10^4$ and we find

$$\gamma_{s,2} = \frac{1}{6.15} - \frac{1}{1.7 \times 10^6} + \frac{1}{2.4 \times 10^{11}} + \dots \cong \frac{1}{6.15} \quad (29)$$

We can assume then that the correct γ for tip No. 2 lies between $\frac{1}{11.5}$ and $\frac{1}{6.15}$ so that the field at the vertex should be

$$\text{Tip No. 2: } 1.2 \times 10^3 V_a < F_o < 2.2 \times 10^3 V_a \text{ volts/cm} \quad (30)$$

where $V_a = V_o = \text{anode potential in volts}$

Similar calculations for tip No. 1, while not so valid due to visible roughnesses on the surface (Fig. 2-a) give

$$\text{Tip No. 1: } 6.4 \times 10^3 V_a < F_o < 9.3 \times 10^3 V_a \text{ volts/cm} \quad (31)$$

$$(a = 1.2 \times 10^{-5}, \theta = 12^\circ, \gamma \text{ lies between } \frac{1}{13} \text{ and } \frac{1}{9})$$

(d) Effect of Penetration of Field into Surface on Field Strength at Tip.

Apker and Taft³ have pointed out that for field emission from photoconductors where the field penetrates the surface appreciably the equipotential surfaces may have significantly lower curvature at the vertex of the tip than the tip itself, resulting in a large reduction of field strength. That this effect is probably unimportant for germanium can be seen from the following.

Consider first the case for very low emission current densities where thermal equilibrium can still be assumed (no IR drop at tip). Then our computations on the displacement of the surface potential by the applied field indicate that for the fields encountered this quantity is never likely to be more than $\sim \frac{1}{e} E_G$ volts for germanium. Now because the tip is not a sphere but includes a supporting base, the field strength drops off with the displacement θ from the vertex, and may reach half its vertex value for typical emitter tips at $\theta = 100^\circ$ (17). Reduction in curvature of the equipotential surfaces near the tip as the applied field is increased then results from the greater penetration of the field at the vertex than at points for $\theta > 0$. Let the tip surface potential at the vertex be changed by $\frac{1}{e} E_G$ by the field. Then the equipotential surface $V(r) = \frac{1}{e} E_G$ just outside will have moved in until it contacts the tip at the vertex with a radial displacement

$$dr \cong \frac{1}{e} \frac{E_G}{F_o} = \frac{1}{e} \frac{E_G}{\gamma V_a/a} \quad (32)$$

while its displacement for $\theta > 0$ will be less than this and will approach 0 for large θ . Since the field at the tip is inversely proportional to the radius of curvature of the equipotential surface contacting it, the relative field decrease in this extreme case cannot be greater than

$$\frac{dF_o}{F_o} \cong - \frac{dr}{a} \cong \frac{E_G}{e\gamma V_a} \quad (33)$$

For typical values $E_G/e = 1$ volt, $\gamma \cong 10$, $V_a = 5000$ volts so that the field strength would not be expected to change at the vertex by more than 1 part in 50,000.

When the surface potential is displaced more drastically than E_G/e , as by an IR drop at the tip in high resistivity material, this effect would get much larger than this.

(c) The Effects of Finite Emission Densities.

Since field emission observations always require finite current densities, we must now see how these are likely to affect the above discussion for germanium.

The essential assumption in the calculation of the energy level displacement at the surface by the external field is that the Fermi level remain constant up to the surface, i. e., that electronic transitions among various energy states are rapid enough to keep the distribution among those states essentially equal to that of maximum probability for complete equilibrium. The disturbance of this distribution at the surface will depend on the rate at which electrons with selected velocities or energies are removed by the emission process. If the number of electrons which penetrate the surface barrier is a small fraction of those incident upon it with approximately the same energy, the disturbance will be small. This fraction is just the barrier penetrability $D(W)$ discussed earlier. Since the computations on surface potential displacement with field for germanium indicated that under equilibrium conditions, the electron

density should always be high enough at the surface at the fields encountered to initiate the emission process while $D(W)$ is still very much less than unity, the disturbance in the distribution should be small for low emission densities.

However, once emission has begun, electrons must be supplied from the interior fast enough to make up for those emitted. If there is a region near the surface of high enough resistivity, an IR drop may be set up and large departures from thermal equilibrium in the surface distribution could result.

We can estimate whether such an IR drop was important for the germanium tips used by multiplying the total emission current I_T by a tip resistance of the form

$$R_{\text{tip}} = \frac{c\rho}{a} \quad (34)$$

where ρ is the resistivity, a the radius and c a geometric factor on the order of 5. Taking ρ as equal to or less than that for intrinsic germanium - approximately 40 ohm-cm - $a \sim 10^{-5}$ cm, and $I_T \sim 10^{-6}$ amp (this value was seldom exceeded in this study), we have

$$I_T R_{\text{tip}} \leq 10^{-6} \frac{5 \cdot 40}{10^{-5}} = 20 \text{ volts} \quad (35)$$

While this is still small enough so that no appreciable alteration in field strength should result from a change in equipotential surfaces by Eq. (33),* on the other hand it is a potential change of approximately 10^3 kT units in a distance of the order of the space-charge thickness, so that appreciable departures from thermal equilibrium might be expected. However, field emission was first noticeable in this study at $I_T \sim 10^{-10}$ amp, where $I_T R_{\text{tip}} \sim 0.002$ volts, or approximately $\frac{1}{10}$ kT unit. At incipient emission, then, we can probably assume our zero emission calculations are justified.

(f) Results on Field Strengths Required for Germanium Field Emission

We are now ready to estimate the surface field strengths which first

* The fact that the emission pattern was never seen to change size as emission current increased confirms the fact that there was no appreciable distortion of the field at the tip by an IR drop.

caused visible emission for tips Nos. 1 and 2. To compare the two, we should find values giving equal emission current density for each. We estimate this density at the tip surface from the relation

$$J = \frac{I_T M^2}{A_{sc}} \text{ amp/cm}^2, \quad (36)$$

where I_T = total emission current in amperes

A_{sc} = approximate area of screen, (radius b), illuminated by emission

M = magnification ratio $\cong b/a$

The chief assumptions involved in (36) are (i) that the emission is fairly uniform over the illuminated spots, and (ii) that the electrons travel on true radial paths to the screen. If there are several spots considered (i) may be off by a factor of ~ 5 , and if there are appreciable surface roughnesses, M in (ii) may be off by a factor of 2.* However, while this may lead to a 20-fold error in J , this would represent an error in the corresponding field strength of only 20 - 30% (see Fig. 3).

We arbitrarily compare the two tips at an incipient emission value of $I_T = 10^{-10}$ amp for tip No. 1, which gives with (36), using $A_{sc} = 0.5 \text{ cm}^2$, $M = 415,000$, a value of $J = 37 \text{ amp/cm}^2$. This value of I_T was reached for tip No. 1 at $V_a = 1,800$ volts, or at a field strength at the tip vertex using (31), of

$$1.2 \times 10^7 < F_o < 1.7 \times 10^7 \text{ volts/cm} \quad (37)$$

For tip No. 2, $M = 68,500$, an emission current density of 37 amp/cm^2 should correspond, for $A_{sc} \sim 0.5 \text{ cm}^2$ in the run considered, to $I_T = 40 \times 10^{-10}$ amp, which occurred at $V_a = 5500$ volts. Equation (30) thus gives the corresponding field strength for tip No. 2 as

$$0.65 \times 10^7 < F_o < 1.21 \times 10^7 \text{ volts/cm} \quad (38)$$

* M will be greater than b/a because of surface protuberances and less because of the displacement of the lines of force toward $\theta = 0$ by the presence of the tip shank.

It is interesting to compare these results with the case of tungsten. Dyke and Trolan¹⁵ measured the field strength required to produce a current density of $\sim 37 \text{ am/cm}^2$ from a clean tungsten tip as between 2.9 and 4.2×10^7 volts/cm, and the Fowler-Nordheim equation, using $\phi = 4.5 \text{ eV}$, gives

$$F_0 = 3.2 \times 10^7 \text{ volts/cm} \quad (39)$$

as the required field for this density. In order to yield a J of 37 amp/cm^2 at F_0 approximately 1.5×10^7 volts/cm, a work function ϕ of between 2.5 and 3.0 eV would have to be used in the Fowler-Nordheim equation.

However, in view of the fact that the emission obtained from the germanium tips was usually in the form of small spots, surface roughness probably made the field somewhat larger than the values in (37) and (38), so that the discrepancy between tungsten and germanium may be less than indicated.

(g) Slope of Field Emission Plot and Absolute Value of Work Function

Fowler and Nordheim^{13, 18} integrated an expression of the form of Eq. (15) to give the field emission of free electrons from a metal with a somewhat idealized boundary. Their result can be expressed as¹⁹

$$J = \frac{e^2 F^2}{8\pi h \phi} e^{-\frac{4}{3} \frac{\sqrt{2m} \phi}{\hbar e} f(y)} \phi^{3/2} \quad (40)$$

where ϕ is the work function, $y = \frac{\sqrt{e^3 F}}{\phi}$ is a measure of the relative Schottky lowering of the potential barrier, and $f(y)$ is a smoothly varying elliptical function of y going from 1 to 0 as y goes from 0 to 1.

Taking logarithms of both sides of (40) we have

$$\ln \frac{J}{F^2} = \ln \frac{e^2}{8\pi h \phi} - \left(\frac{4}{3} \frac{\sqrt{2m}}{\hbar e} f(y) \right) \frac{\phi^{3/2}}{F} \quad (41)$$

Since $f(y)$ remains close to unity over most of the F values normally encountered, a plot of $\ln \frac{J}{F^2}$ vs $\frac{1}{F}$ should give a straight line of slope $\approx -\frac{4}{3} \frac{\sqrt{2m}}{\hbar e} \phi^{3/2}$. In practice F^2 varies so slowly compared to J that

practically no error results in taking the slope of the $\ln J$ vs $\frac{1}{F}$ plot. If the free electron mass is used for m in Eq (40) and J is in $\frac{\text{amp}}{\text{cm}^2}$, F in volts/cm, we find

$$\phi \cong 6.0 \times 10^{-6} \left(\text{slope } \ln J \text{ vs } \frac{1}{F} \right)^{\frac{2}{3}} \text{ eV} \quad (42)$$

Measured values of this slope from data on tips No. 1 and 2 ranged from 1 to 2×10^8 , giving indicated work functions of

$$\phi = 1.8 \text{ to } 2.1 \text{ eV.} \quad (43)$$

These results, which are significantly lower than the work function of germanium (4.5 to 4.8 eV), are hard to explain in terms of the simple treatment given above. The greatest decrease in ϕ that could result from the lowering of the levels at the surface would be on the order of E_G , but this should still leave $\phi \cong 4.7 - 0.7 = 4.0$ eV. The value of the slope of the $\ln J$ vs $\frac{1}{F}$ plot is insensitive to absolute errors made in measuring J , and the relative values measured for J should be accurate to within 20 per cent. Errors in the computed field show up directly as errors in the slope, but it would take a roughness factor enhancing the field above the computed value of $\left(\frac{4.7^3}{2.0}\right)^{1/2} \cong 3.7$ to bring the indicated work function up to 4.7 eV. While it seems doubtful that such a large roughness factor is possible, considering the smooth profile of tip No. 2 as seen both before and after the study in the electron microscope pictures, it must be noted that about this same value would bring the incipient emission field values for germanium, (Eqs. (37), (38)) into close correspondence with that for tungsten.*

If the calculated fields and slopes are correct, it is possible that the quantum-mechanical derivations used for the metal, (Fowler-Nordheim equations) are not sufficiently valid as applied to the semiconductor. However, it is felt that since the barrier outside the surface is very similar for both metal and semiconductor, and since this is the controlling factor in field emission, the Fowler-Nordheim equations should apply fairly well.

*One possibility that has not been mentioned is that most of the emission studied originated from small clusters of adsorbed molecules of low work function on the germanium surface. However, this seems doubtful since the emission observed, for example, Fig. 4, appeared to originate from large regions of the smooth tip surface itself.

A progressive downward displacement of the energy levels at the surface with respect to the Fermi level as the field increases should increase the slope of the plot. However, since the field changes relatively little during any run, this effect would be small for the data taken.

(h) Reasons for Failure to get Good Emission Patterns.

One possible explanation for the difficulty experienced in finding symmetrical emission patterns from germanium lies in the low values of work function differences measured by the C.P.D. method on different germanium crystal faces and reported in Technical Report 236. Thus, the differences of approximately 0.06 eV between the (100) and (111) and the 0.015 eV between the (110) and (111) plane measured for germanium are to be compared with the corresponding differences measured for tungsten of 0.13 and ~ 1.0 eV.^{20, 21}

The ratio of emission current densities at a given field strength for two regions of a tip differing in work function by $\Delta\phi$ is given by the Fowler-Nordheim equation (40) to be (noting that $f(y)$ is a slowly varying function of ϕ):

$$\frac{J(F, \phi)}{J(F, \phi + \Delta\phi)} = e^{\frac{6.85 \times 10^7 f(y)}{F} \left[\left(1 + \frac{\Delta\phi}{\phi}\right)^{\frac{3}{2}} - 1 \right] \phi^{\frac{3}{2}}} \quad (44)$$

To illustrate Eq. (44), Table I shows values for the current density ratio computed for two values of $\frac{\Delta\phi}{\phi}$ and F .

Table I

Ratio of emission current densities for different work functions from the Fowler-Nordheim equation.

$\Delta\phi$ eV	$\frac{\Delta\phi}{\phi}$	$\frac{J(F, \phi)}{J(F, \phi + \Delta\phi)}$
1. F	$= 10^7$ volts/cm;	$\phi = 4.5$ eV
0.045	.01	2.4/1
0.45	.1	9000/1
2. F	$= 3 \times 10^7$ volts/cm;	$\phi = 4.5$ eV
0.045	.01	1.3/1
0.45	.1	12/1

We see that the ratios are largest at low field strengths, but that even at the lowest field strengths likely to be met, 1×10^7 volts/cm, the differences in ϕ found for germanium would provide extremely weak intensity contrast compared to those found in the tungsten emission patterns.

Another possible reason for the failure to get good emission patterns is that if the energy levels at the surface are depressed appreciably by the field, the differences in ϕ measured by the C.P.D. method at zero fields - already small - may have been even further reduced by the applied field.

Acknowledgements

The author wishes to acknowledge the support of this study by the Office of Naval Research, the Signal Corps of the U. S. Army and of the U. S. Air Force. Particular thanks are due Professor E. L. Chaffee, who suggested the project described in this report, and Professor Harvey Brooks, my advisor since Professor Chaffee's retirement. In addition, I have benefited from discussions with people at the General Electric Research Laboratory in Schenectady, and from both discussions and samples from Raytheon Manufacturing Company in Waltham, Massachusetts, and from the Philco Corporation, Philadelphia, Pennsylvania. Use of the M.I.T. electron microscope was made possible through the kindness of Dr. R. H. Kingston of Lincoln Laboratory.

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